

## \* Combined stresses .

13/10/2015  
مراجعة

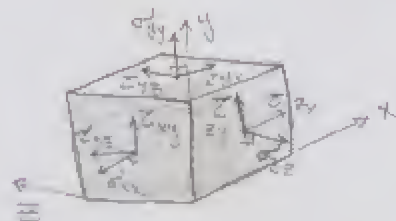
MR  
AY



$$\tau_{xy} = \tau_{yx} \quad \sigma_{xx} = \sigma_{xx}$$

$$\tau_{yz} = \tau_{zy} \quad \sigma_{yy} = \sigma_{yy}$$

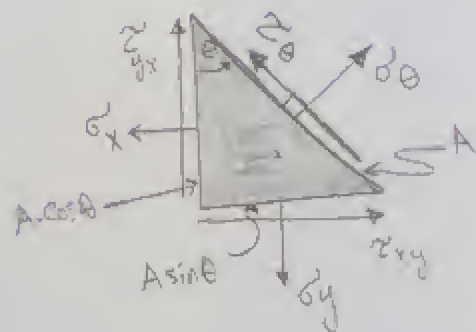
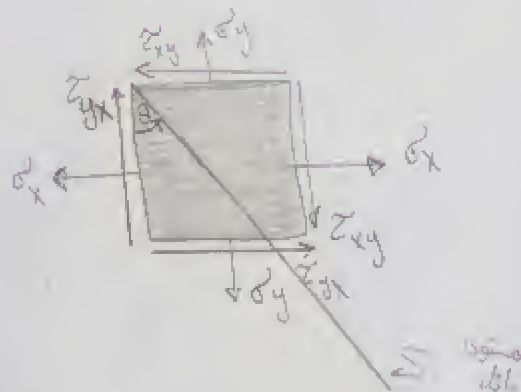
$$\tau_{zx} = \tau_{xz} \quad \sigma_{zz} = \sigma_{zz}$$



tensor

$\sigma_{xx}$	$\tau_{xy}$	$\tau_{xz}$
$\tau_{yx}$	$\sigma_{yy}$	$\tau_{yz}$
$\tau_{zx}$	$\tau_{zy}$	$\sigma_{zz}$

## \* plan stresses .



$$\sigma_{\theta} \cdot A = \sigma_x \cdot A \cos \theta \cdot \cos \theta + \sigma_y \cdot A \sin \theta \cdot \sin \theta + \tau_{xy} \cdot A \sin \theta \cos \theta + \tau_{yx} \cdot A \sin \theta \cdot \cos \theta$$

$$\sigma_{\theta} = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cdot \cos \theta +$$

1b)

$$\sin^2 \theta = \frac{1}{2} (1 - \cos 2\theta)$$

$$\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$$

$$\sigma_{\theta} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \cdot \sin 2\theta \rightarrow (1)$$

$$\tau_{\theta} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cdot \cos 2\theta \rightarrow (2)$$

$$\frac{d\sigma_{\theta}}{d\theta} = -(\sigma_x - \sigma_y) \cdot \sin 2\theta + 2\tau_{xy} \cdot \cos 2\theta = 0$$

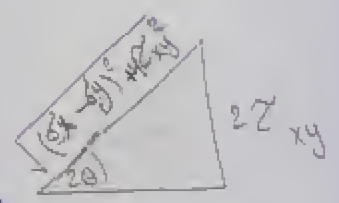
• مت تلك  
• العلاقة

$$\frac{d\sigma_{\theta}}{d\theta} = 2\tau_{\theta} = 0$$

$$\tau_{\theta} = 0$$

الذي ماله  $\tau_{\theta} \leftarrow$  في حالة المستوى أكبر باجناد عمودي تكون قيمها يساوي صفر. Note.

$$\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)} \rightarrow (3)$$



$$\sigma_{1,2} = \frac{(\sigma_x + \sigma_y)}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \rightarrow (4)$$

$$\sigma_1 = \max \quad \sigma_2 = \min$$

في حالة المستوى أكبر باجناد عمودي تكون قيمة  $\sigma_{\theta}$  يساوي  $\frac{\sigma_x + \sigma_y}{2}$  التي ماله

$$\frac{d\tau_{\theta}}{d\theta} = 0$$

$$\tan 2\theta = \frac{-(\sigma_x - \sigma_y)}{2\tau_{xy}} \rightarrow (5)$$

$$\tau_{\max} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \rightarrow \boxed{6}$$